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Prediction of Vortex Breakdown and Longitudinal Characteristics of Swept Slender Planforms

Lance W. Traub*
Texas A&M University, College Station, Texas 77843-3141

A method is presented to predict high-angle-of-attack, longitudinal aerodynamic characteristics of slender wing planforms in incompressible flow. A semiempirical approach is used to predict the location of vortex breakdown and its variation with incidence. Breakdown predictions are then coupled to vortex lift expressions based on the leading-edge suction analogy. A correction is used to account for the attenuation of vortex suction after vortex breakdown, allowing prediction of lift, drag, and pitching moment at high angles of attack. Comparisons are made with a variety of planforms, with encouraging agreement between theory and experiment being demonstrated.

Nomenclature

b = wing span
C_D = drag coefficient
C_L = lift coefficient
C_m = pitching moment coefficient
c = local chord
c_L = section lift coefficient
c_r = root chord
k = residual vortex lift parameter, Eq. (12)
k_p = potential constant
P = Squires' planform parameter
S = wing area
U_∞ = freestream velocity
x = chordwise direction
α = angle of attack
Γ = vortex circulation
$\Gamma_{\alpha>\text{BD-TE}}$ = circulation at trailing edge at angles of attack beyond that at which breakdown first occurs at the trailing edge
ε = wing apex half-angle
η = nondimensional spanwise coordinate
Λ = leading-edge sweep angle

Subscripts
BD = breakdown
BD-TE = breakdown at wing trailing edge
ca = centroid of planform area
ca1 = centroid of area from wing apex to location of vortex breakdown
ca2 = centroid of area from vortex breakdown location to wing trailing edge
max = maximum
p = potential flow
VBD = vortex lift developed over wing from wing apex to the location of vortex breakdown
VLE = leading-edge vortex lift
VNB = vortex lift for the whole planform assuming no vortex breakdown

Introduction

SUPERMANEUVERABILITY encompasses routine operation of a fighter-type aircraft at high angles of attack. These types of aircraft typically have wings with a swept leading edge, which may be combined with a highly swept strake or forebody chine, forming a strake–wing. At moderate to high α these configurations benefit from the lift-enhancing effects of the vortices generated by the roll up of the shear layer emanating from the strake/wing's leading edge. The vortex lift produced by these leading-edge vortices greatly enhances the lift of slender wings. The vortices are also generally very stable. However, at high angles of attack these vortices may burst, a phenomenon associated with a rapid deceleration of the vortex core and an increase in the diameter of the resulting structure. Vortex breakdown (BD) may essentially be related to the vortex having insufficient axial flow to convect axial vorticity downstream, such that a threshold level may be exceeded.[1] BD affects the aerodynamic characteristics of the wing by reducing the magnitude of the vortex suction peak and increasing its width. This in turn, depending on the wing leading-edge sweep, may reduce suction levels over the rear of the wing and cause an undesirable nose-up pitching moment. Periodic phenomena in the wake-type flow of the burst vortex can also introduce fatigue of aft surfaces through periodic pressure fluctuations.[2]

Although accurate simple prediction methods exist (e.g., Polhamus'[3] suction analogy) to determine the characteristics of slender wings, they are limited by the onset of BD. Lan and Hsu[4] developed empirical expressions to account for BD, which were then coupled with a vortex lattice code to predict the aerodynamic characteristics of slender wings in the presence of vortex breakdown. The empirical data to account for BD was based on the results of Wentz and Kohlman.[5] This approach would generally limit the applicability of the method to planforms for which Wentz and Kohlman's results[5] for delta and double delta wings are representative. Estimation of C_L at high α for moderately swept wings is also not particularly accurate. Brandt,[6,7] using the incompressible Navier–Stokes equations, derived a model for the vortex core, allowing its incorporation into a vortex lattice method, which then facilitated the prediction of vortex breakdown. Studies have shown that even after BD, substantial vortex suction levels are still generated.[8,9] To account for this, Brandt[6,7] scaled the vortex lift by the ratio of unburst vortex length to total length. However, for the cases cited,[7] the method does not apparently provide accurate estimates of C_L beyond $C_{L_{max}}$. Prediction of C_m also does not appear to be consistent with the experimental data. Euler and Navier–Stokes solvers are capable of predicting high α performance, but are computationally expensive. These

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*Graduate Student, Aerospace Engineering Department. Associate Member AIAA.

solvers are generally not feasible for parameter variation or conceptual design studies.

In this paper, a simple incompressible analytic prediction method is presented that allows estimation at high α of lift, drag, and pitching moment for slender swept planforms. The suction analogy coupled with a BD prediction method is used in combination with an empirical correction to account for the reduction in suction under the burst vortex. Comparisons between theory and experiment are presented.

Discussion of Method

Vortex Breakdown

It is well known that the breakdown of leading-edge vortices is strongly influenced by two factors: 1) the swirl angle[10] of the vortex, i.e., the ratio of the rotary to axial velocity components, and 2) the imposed adverse pressure gradient. This pressure gradient may be considered to consist of two components. The first is due to the growth of the vortex core resulting from vorticity diffusion, and the second, the pressure gradient associated with enforcement of the Kutta condition at the wing trailing edge and its associated pressure recovery. This external pressure gradient is amplified[8] because of flow swirl and transmitted to the vortex core via the radial pressure balance.[6] Studies[10] have shown that breakdown is typically associated with swirl angles ranging from 45 to 55 deg. An Euler and Navier–Stokes[11] analysis of vortex breakdown to substantiate the Rossby number found that BD occurred at Rossby numbers, i.e., the ratio of the axial velocity to the maximum rotary velocity at the edge of the viscous core, below the range 0.9–1.4. It is also usually assumed that the maximum swirl angle is associated with the flow at the edge of the viscous core; however, Hawk et al.[12] have shown that the maximum swirl angle is confined to the leading-edge shear layer. Thus, although criteria do exist to predict the onset of BD, they require accurate prediction of the axial and rotational velocity profiles. A simple analytic method to predict BD would not be amenable to estimating velocity components.

The following breakdown methodology is similar to that presented in a previous paper[13]; however, accuracy has been improved by using a different criterion to account for the reduction of circulation at BD, and the method has been extended to a wider range of planforms; the procedure of Ref. 13 is limited to pure delta wings. Conical flow theory suggests that the vortex circulation increases linearly with distance from the wing apex. Experimental[1] and Euler[14] studies have shown this to be correct, except near the wing trailing edge, where circulation increases at a decreasing rate. Thus, the chordwise variation of circulation may be written as

$$\Gamma / U_\infty c_r = A(x/c_r) \tag{1}$$

where A is a constant of proportionality. Hemsch and Luckring,[15] using Sychev similarity parameters, estimated the nondimensional strength or circulation of the leading-edge vortex at the wing trailing edge as

$$\Gamma / U_\infty c_r = 4.63 \tan^{0.8}\varepsilon \tan^{1.2}\alpha \cos\alpha \tag{2}$$

where the constant 4.63 was suggested by Visser and Nelson[1] to correlate their test results. The location of $(x/c_r)_{\mathrm{BD}}$ with increasing α, when BD has progressed forward of the wing's trailing edge ($\alpha > \alpha_{\mathrm{BD\text{-}TE}}$), may be initially estimated by assuming that at the breakdown location, the nondimensional circulation, $\Gamma_{\mathrm{BD}}/U_\infty c_r$, is constant and equal to the value when breakdown first occurs at the trailing edge, i.e., $\Gamma_{\mathrm{BD}}/U_\infty c_r = \Gamma_{\mathrm{BD\text{-}TE}}/U_\infty c_r$. Then, linearly interpolating between the circulation at the wing apex, which is equal to 0, as shown by Eq. (1) and the wing trailing edge ($\Gamma_{\alpha > \mathrm{BD\text{-}TE}}/U_\infty c_r$) to find where the circulation is equal to $\Gamma_{\mathrm{BD\text{-}TE}}/U_\infty c_r$, yields[13]

$$\left(\frac{x}{c_r}\right)_{\mathrm{BD}} = \frac{\Gamma_{\mathrm{BD\text{-}TE}}}{\Gamma_{\alpha > \mathrm{BD\text{-}TE}}} \tag{3}$$

This expression assumes that when BD occurs over the wing, circulation is constant, and equal to that when BD first occurs at the trailing edge. This assumption of constant circulation is not correct. As the angle of attack of the wing is increased and the location of breakdown moves forward along the chord, the circulation is reduced. The circulation will approach zero as breakdown approaches the wing apex. A suitable empirical correction to account for this is of the form

$$\frac{\Gamma_{\mathrm{BD}}}{U_\infty c_r} = \frac{\Gamma_{\mathrm{BD\text{-}TE}}}{U_\infty c_r}\left(\frac{x}{c_r}\right)_{\mathrm{BD}}^{1/2} \tag{4}$$

Thus Eq. (3) may be rewritten as

$$\left(\frac{x}{c_r}\right)_{\mathrm{BD}} = \frac{\Gamma_{\mathrm{BD}}}{\Gamma_{\alpha > \mathrm{BD\text{-}TE}}} \tag{5}$$

To calculate the progression of BD from the wing's trailing edge to the apex, the numerator and denominator of Eq. (5) are calculated using Eq. (2). Equation (4) and (5) [and thus Eq. (2)] would then be iterated while increasing α until breakdown reaches the apex. Note that the $\Gamma_{\mathrm{BD}}/U_\infty c_r$ used in Eq. (5) is determined at the previous $(x/c_r)_{\mathrm{BD}}$ location to that being calculated. The predicted movement of $(x/c)_{\mathrm{BD}}$ is not particularly sensitive to the incremental α used in iterating, provided the incremental $\alpha \leq 2$ deg. As will be shown later, this method generally shows excellent correlation with experimental results, and suggests that the strength of the vortex at the breakdown location is related to the square root of the distance of breakdown from the apex. Straka and Hemsch[16] conducted a comprehensive experimental study of the effect of wing planform on BD for wings with the same slenderness ratio. It was found that if the results were plotted as a function of local semispan, they showed a tendency to collapse on each other for wings with a Squires planform parameter of less than one-half. This planform parameter is defined[16] as

$$P = \text{exposed planform area}/c_r b$$

where $P = 0.5$ corresponds to a delta wing. This observation allows for an approximate prediction of BD location for arbitrary planforms, provided $P \leq 0.5$, i.e., no gothic wings. Figure 1 shows that if the variation of breakdown location with α for the equivalent slenderness ratio, i.e., $b/2c_r$, delta to the wing being studied can be found, then breakdown on the actual planform can be determined. A flow chart detailing this process is included in Fig. 1. The method detailed earlier in this paper can be used to estimate the location of breakdown on an arbitrary planform slender wing, provided $P \leq 0.5$.

To estimate $\alpha_{\mathrm{BD\text{-}TE}}$, the following equation from Ref. 13 may be used. Although it was derived for pure deltas, Straka and Hemsch[16] found that wings with a common slenderness ratio typically experience initial BD at the trailing edge at similar angles of attack. Alternatively, experimental data may be used:

$$\alpha_{\mathrm{BD\text{-}TE}} = \tan^{-1}(13.47 \tan\varepsilon\, e^{-6.9\varepsilon}) \tag{6}$$

Longitudinal Aerodynamic Coefficients

In Polhamus'[3] suction analogy, lift is decomposed into two components, one due to attached or potential flow and one due to vortex lift. The attached flow component was formulated by Polhamus,[3] as the attached flow lift experienced by the wing in the absence of any leading-edge suction force. The vortex lift is modeled through a 90-deg rotation of the leading-edge suction force to supplement the normal force coefficient. Total lift is then given by

$$C_L = C_{L_p} + C_{L_{\mathrm{VLE}}} \tag{7}$$

and the potential lift

$$C_{L_p} = k_p \sin\alpha \cos^2\alpha \tag{8}$$

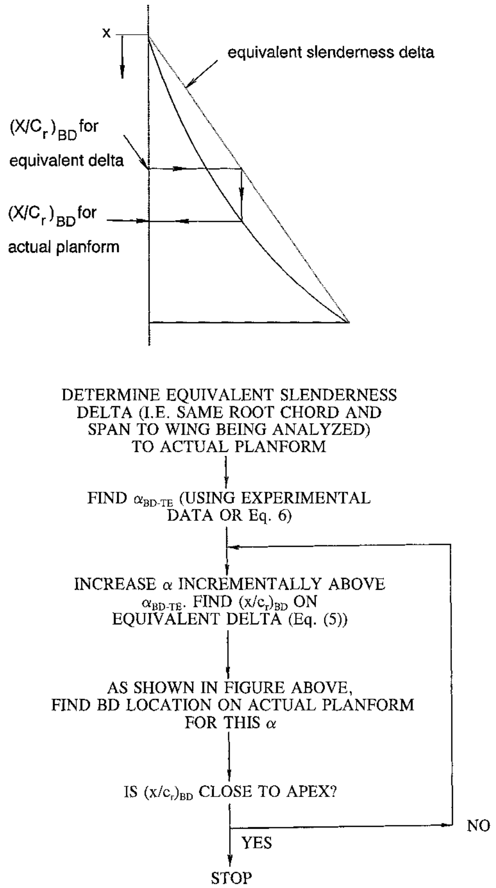

Fig. 1 Method to determine location of vortex breakdown on arbitrary planforms, $P < 0.5$.

Purvis[17] suggested a method, based on the suction analogy, and used a prescribed pressure distribution to predict C_L, C_D, and C_m of arbitrary planforms. He gives an expression for the local leading-edge vortex lift for an arbitrary planform as

$$cc_{L_{\mathrm{VLE}}} = \frac{1.106 C_k S}{b \cos \Lambda} \left(\eta \sqrt{1 - \eta^2} + \arcsin \eta\right) \tag{9}$$

where

$$C_k = k_p \sin^2\alpha \cos^3\alpha [1 - (k_p/\pi \mathrm{AR}) \cos^3\alpha] \tag{10}$$

k_p is the lift-curve slope at $C_L = 0$, and may be determined using lifting surface theory. Spanwise integration of Eq. (9) gives

$$C_{L_{\mathrm{VLE}}} = 1.106 C_k \left[\frac{-\frac{1}{3}(1 - \eta^2)^{3/2} + \eta \arcsin \eta + \sqrt{1 - \eta^2}}{\cos \Lambda} \right]_{\eta_1}^{\eta_2} \tag{11}$$

The total vortex lift, assuming no breakdown effects would be given by Eq. (11) by setting the integration limits: $\eta_2 = 1$ and $\eta_1 = 0$. If the leading-edge sweep angle varies, then $\Lambda = \Lambda(\eta)$ and Eq. (11) is evaluated using the respective Λ between the appropriate limits.

Estimation of the effect of BD on the aerodynamic coefficients will unavoidably require some simplifying assumptions. Generally, studies[18,19] have shown that the effect of BD on C_L is very much dependent on the leading-edge sweep angle of the planform. For relatively low wing sweep (60 deg), vortex breakdown has a marginal effect on lift or lift-curve slope, until breakdown approaches the wing apex. Er-El and Yitzhak[9] show that vortex lift is only affected when breakdown is in the vicinity of the apex. For wings with leading-edge sweep greater than 70 deg, vortex breakdown at the trailing edge has a more immediate effect on lift. For a 75-deg sweep delta wing Er-El and Yitzhak[9] show that the initial onset of BD drastically reduces vortex lift. These contrary and nonuniform characteristics complicate the prediction methodology.

In the present method it is assumed that the vortex lift is unaffected by breakdown upstream of the BD location (Lambourne and Bryer[8] have shown that BD does have a moderate effect on the upstream surface pressures), and that from the location of BD to the trailing edge the residual vortex suction is a function of α only, and is distributed uniformly over the surface. This assumption may, in part, be justified by noting that the wing surface pressure distribution in the BD region does appear to partially flatten out[18] in a manner analogous to that seen in wake or separated flow, where the pressure distribution is often assumed to be constant. As noted in Ref. 13 for delta wings with leading-edge sweep angles from 65 to 80 deg, BD typically takes between 20–30 deg to progress from the wing's trailing edge to the apex. Consequently, the residual vortex suction level is assumed to reduce linearly, from a maximum at BD onset, to zero at an additional 30 deg α, as follows:

$$k = 1 - (\alpha - \alpha_{\mathrm{BD\text{-}TE}})/30 \tag{12}$$

For $\alpha < \alpha_{\mathrm{BD\text{-}TE}}$, k is set to 1.

The total vortex lift is then given by

$$C_{L_{\mathrm{VLE}}} = C_{L_{\mathrm{VBD}}} + k(C_{L_{\mathrm{VNB}}} - C_{L_{\mathrm{VBD}}}) \tag{13}$$

where the first term in the parentheses is the vortex lift for the whole planform assuming no breakdown, and the second term in the parentheses is the vortex lift developed from the wing apex to the location of vortex breakdown. Both of these terms can be evaluated using Eq. (11), with suitable integration limits, which in the latter case are a function of $(x/c_r)_{\mathrm{BD}}$. For a double delta, Eq. (36) of Ref. 17 may be used to find $C_{L_{\mathrm{VNB}}}$.

In the analysis presented, breakdown and its progression has no effect on the attached flow lift contribution. Er-El and Yitzhak's[9] results show that depending on the wing leading-edge sweep, potential lift may be over- or underpredicted; however, BD doesn't appear to have any obvious effect on the potential lift coefficient.

The suction analogy gives overall prediction of lift and drag, and not their distribution, thus generally limiting its suitability in predicting pitching moment. However, pitching moment may be calculated using a formulation similar to that of Bartlett and Vidal,[20] based on crossflow theory:

$$C_m = \frac{\mathrm{d}C_m}{\mathrm{d}C_{L_p}} C_{L_p} - \frac{x_{\mathrm{ca}}}{c_r} C_{L_{\mathrm{VNB}}} \tag{14}$$

In this expression, the vortex lift acts at the centroid of the planform. Generally BD results in a loss of suction progressing from the trailing edge to the apex. This results in a destabilizing pitching moment. To incorporate BD effects into the pitching moment calculations, it is assumed that the potential flow contribution to the pitching moment is unaffected by breakdown. The moment as a result of vortex lift is then calculated as consisting of a component caused by full vortex suction from the apex to $(x/c_r)_{\mathrm{BD}}$, and a component caused by the attenuated suction from $(x/c_r)_{\mathrm{BD}}$ to the trailing edge. In each region the total suction is assumed to act at the centroid of the respective area, yielding

$$C_m = \frac{\mathrm{d}C_m}{\mathrm{d}C_{L_p}} C_{L_p} - \frac{x_{\mathrm{ca2}}}{c_r} k(C_{L_{\mathrm{VNB}}} - C_{L_{\mathrm{VBD}}}) - \frac{x_{\mathrm{ca1}}}{c_r} C_{L_{\mathrm{VBD}}} \tag{15}$$

Equation (15) gives the pitching moment about the wing apex, nondimensionalized by the wing root chord.

Comparison with Experiment

For comparisons of theory with experiment for vortex burst trajectories, experimental data for $\alpha_{BD\text{-}TE}$ were used. Figures 2–5 show predictions for the location of vortex breakdown over various wing planforms. Agreement is seen to be very good and to be consistent with the results of Erickson.[22] The accurate prediction of the breakdown location for the concave planform and the double delta is especially encouraging and verifies the observations of Straka and Hemsch.[16] Figures 2 and 3 also include vortex burst trajectories determined using the method of Ref. 13. Correlation between the two methods for the 70-deg leading-edge sweep delta wing is good.[23] For the 60-deg sweep delta, however, the present method shows closer accord with the experimental data. As may be seen in Figs. 2–5, there is a decrease in the rate of progression of BD toward the wing apex, once breakdown has passed approximately $x/c_r = 0.5$. This is due to the flow behaving in a conical fashion forward of midchord, lessening the adverse pressure gradient.

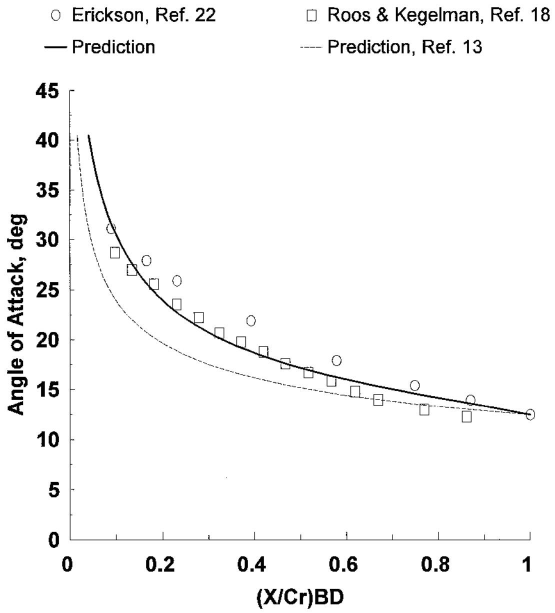

Fig. 2 Vortex burst trajectory for Λ = 60-deg delta wing.

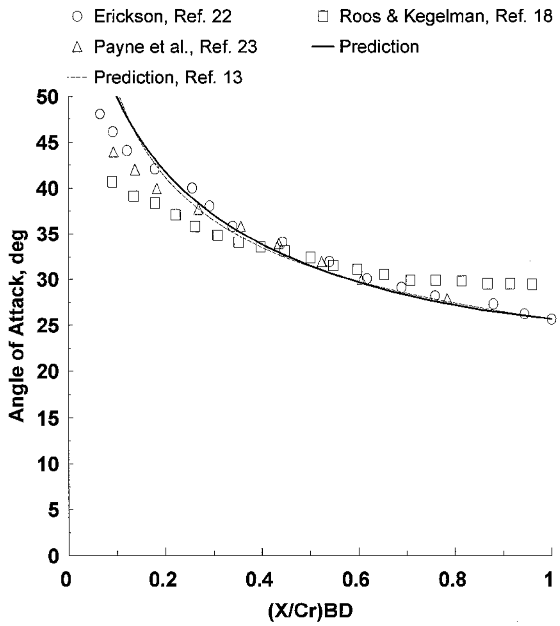

Fig. 3 Vortex burst trajectory for Λ = 70-deg delta wing.

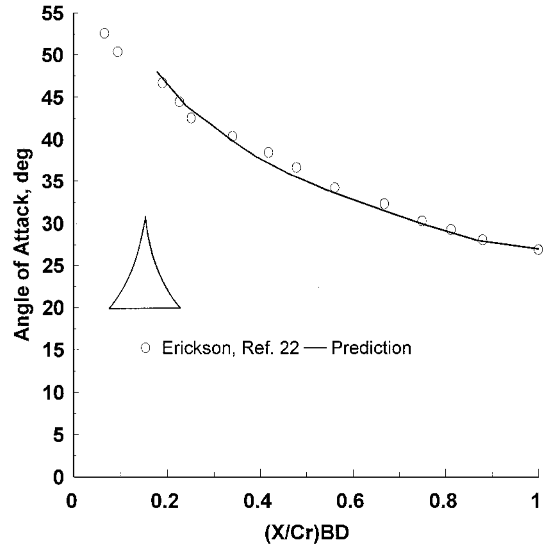

Fig. 4 Vortex burst trajectory for concave wing.

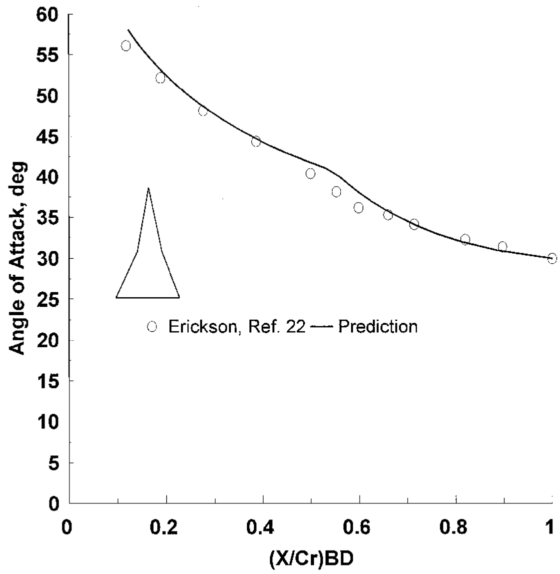

Fig. 5 Vortex burst trajectory for 80-/65-deg double delta wing.

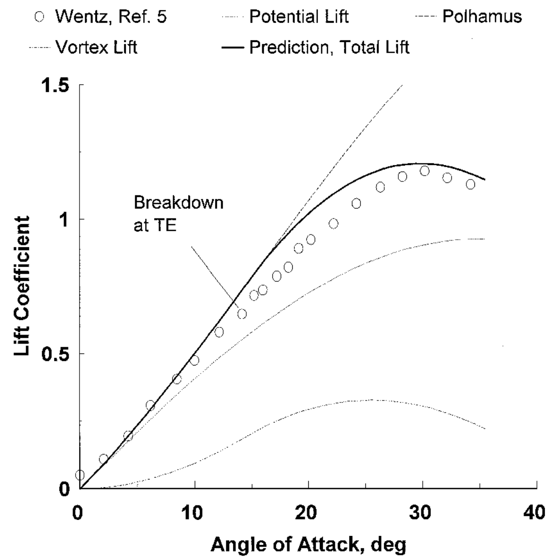

Fig. 6 Comparison of theory and experiment for Λ = 60-deg delta wing.

A complicating factor when validating a prediction method is the selection of representative experimental data. Kegelman and Roos[21] have shown that for slender deltas the wing leading-edge shape and form of beveling has a substantial effect on the maximum lift coefficient. The results of Wentz and Kohlman[5] are often regarded as baseline data, and as most of the test models had symmetrical beveling, they will be used for comparison in this paper. Other representative data have also been included for some configurations. As noted by Wentz and Kohlman,[5] results for the more highly swept wings (sweep > 80 deg) may not be representative because of model flexing, and thus no comparisons are made for these wings.

Figure 6 shows that for a 60-deg sweep delta wing, lift is generally overestimated beyond approximately 10-deg α. Although this effect is sometimes attributed to an overprediction of vortex lift, it is in fact due to the potential lift contribution being overpredicted.[9] $C_{L_{max}}$ is seen to be well estimated. Figure 7 shows lift prediction for a leading-edge sweep of 70 deg. Agreement with the experimental data is seen to be very good

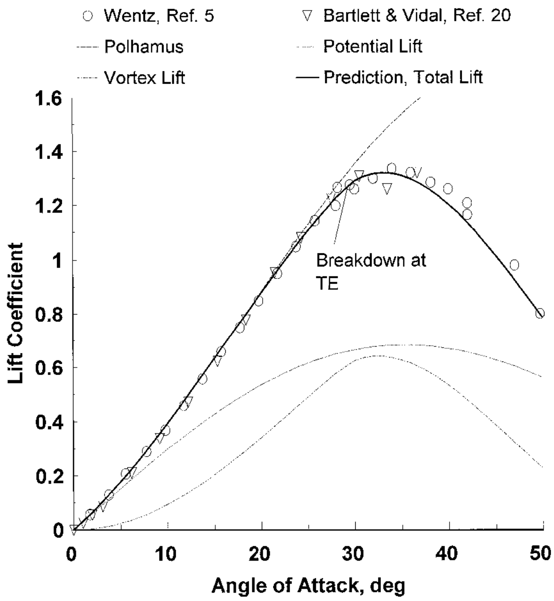

Fig. 7 Comparison of theory and experiment for Λ = 70-deg delta wing.

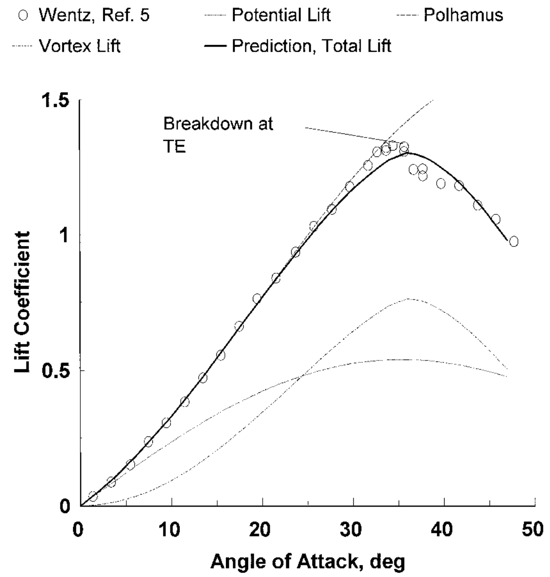

Fig. 8 Comparison of theory and experiment for Λ = 75-deg delta wing.

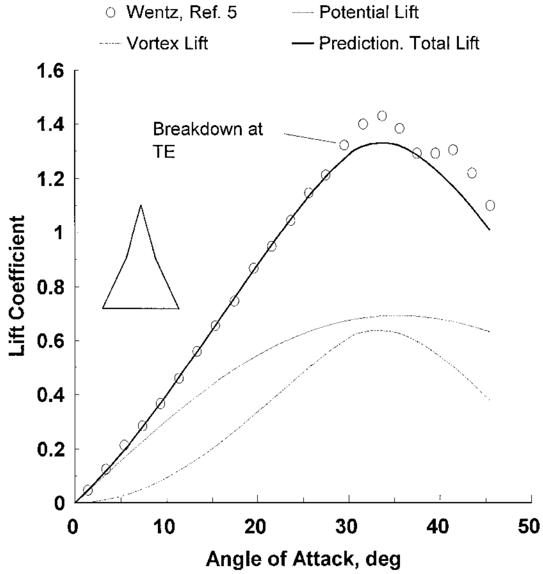

Fig. 9 Comparison of theory and experiment for 75-/65-deg double delta wing.

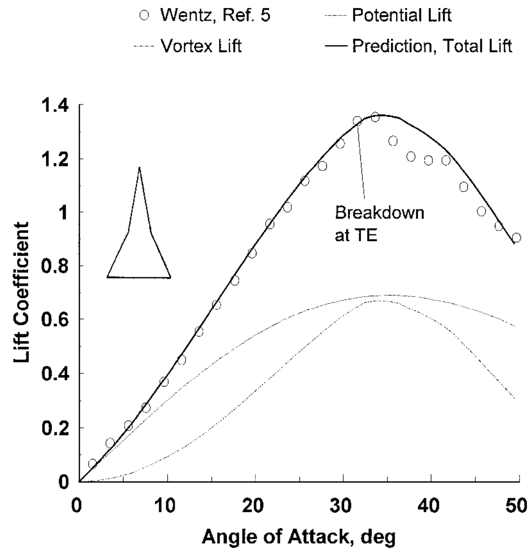

Fig. 10 Comparison of theory and experiment for 80-/65-deg double delta wing.

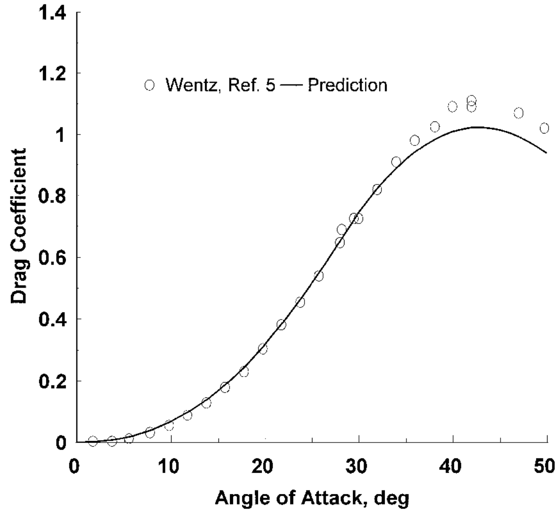

Fig. 11 Comparison of theory and experiment for Λ = 70-deg delta wing.

up to high α. For a sweep angle of 75 deg (Fig. 8), the form of the $C_{L_{max}}$ peak is not as well represented, but agreement at high α is excellent. Figures 9 and 10 show comparisons with 75/65 deg and 80/65 deg sweep double deltas, respectively.

For thin planar wings, inviscid drag is given simply by $C_L \tan(\alpha)$. Figures 11 and 12 show favorable comparisons with the experimental data using this formula. Note that the experimental data have the zero-lift drag coefficient removed. The underprediction of drag at high C_L (Fig. 11) is also seen in comparisons of Wentz and Kohlman,[5] and is due to an increase in parasite drag at high α. The excellent agreement shown in Fig. 12 at α > 40 deg may be somewhat fortuitous and a result of a slight overprediction of lift in this α range.

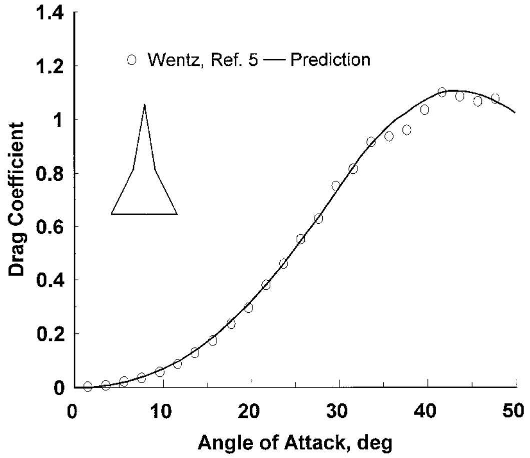

Fig. 12 Comparison of theory and experiment for 80-/65-deg double delta wing.

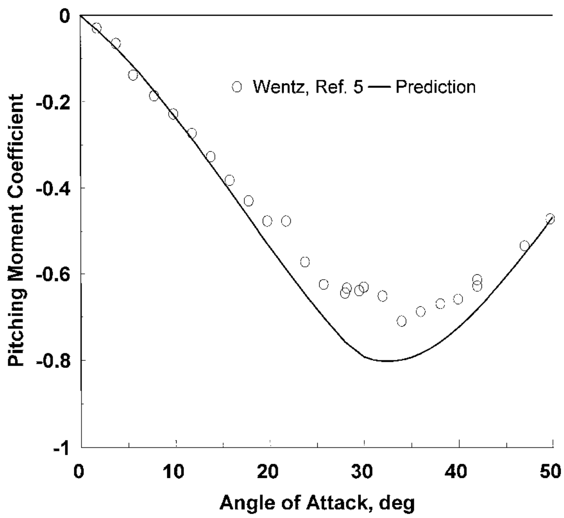

Fig. 13 Comparison of theory and experiment for Λ = 70-deg delta wing.

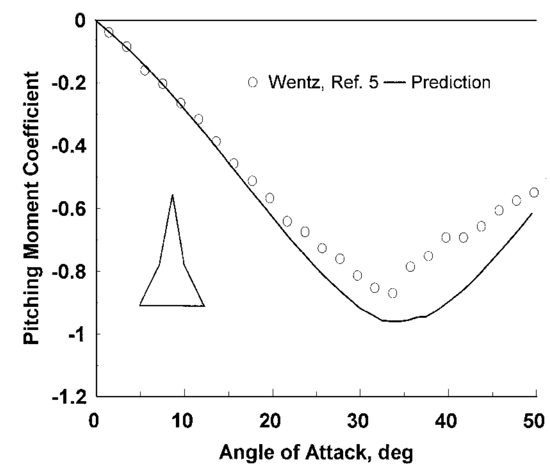

Fig. 14 Comparison of theory and experiment for 80-/65-deg double delta wing.

Figures 13 and 14 present predictions of pitching moment as a function of angle of attack. At high angles of attack, the magnitude of the pitching moment is seen to be moderately overpredicted; however, the characteristic pitch-up is well represented. The moment curves suggest that the prediction method, by assuming a uniform loading concentrated at the respective centroid, places the loading too far aft. This would be expected, as loading in the presence of BD generally decreases toward the rear of the wing.

Concluding Remarks

An incompressible semiempirical method is presented that predicts the location of vortex breakdown, as well as lift, drag, and pitching moment for a variety of planforms. The method allows determination of these characteristics, suitable for conceptual design, up to high angles of attack. It must be noted, however, that the methodology is based upon several simplifying assumptions, described previously, which may limit its application to arbitrary shapes.

Vortex breakdown location was predicted using an interpolation equation, modified to account for breakdown effects. This expression was formulated using a Sychev similarity parameter, and conical flow assumptions. Aerodynamic coefficients were determined using the leading-edge suction analogy, with an empirical modification to account for the attenuation of vortex suction in the presence of vortex breakdown. The method was compared to experimental results, with very good accuracy being demonstrated.

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